

# Hilltop Curvatons

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## Abstract

We study “hilltop” curvatons that evolve on a convex potential. Hilltop curvatons evolving on the Hubble-induced potential are generic if supergravity is assumed in the theory. We do not consider curvatons whose potential is protected from  $O(H)$  corrections, where  $H$  is the Hubble parameter. We assume that the effective mass of a curvaton is expressed as  $m_\sigma = cH$ , where the coefficient varies within  $0.2 \leq c \leq 5$  depending on the circumstances. A negative mass term may lead to enhancement of curvaton fluctuation, which has a significant impact on the energy bound for low-scale inflation. Using a simple curvaton model and following the conventional curvaton hypothesis, we demonstrate the generality of this enhancement.

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# 1 Introduction

The “curvaton” is an alternative formulation for generating curvature perturbation that does not require fluctuation of the inflaton field[1]. The curvaton model has several characteristic features, some of which distinguish curvaton-induced fluctuation from inflaton fluctuation.

In this section we briefly review the calculations related to the lower bound of the inflation scale as derived by Lyth in Ref.[2]. In this paper we consider a significant relaxation of the bound. We initially assume that a curvaton ( $\sigma$ ) is frozen at  $\sigma = \sigma_{osc}$  during and after inflation. At the beginning of curvaton oscillation, the curvaton density is given by  $\rho_\sigma \sim m_\sigma^2 \sigma_{osc}^2 / 2$ . Denoting the total density of all other fields by  $\rho_{tot}$ , the ratio  $r$  at the beginning of curvaton oscillation is

$$r(t_{osc}) = \left. \frac{\rho_\sigma}{\rho_{tot}} \right|_{osc} \sim \frac{m_\sigma^2 \sigma_{osc}^2}{H_{osc}^2 M_p^2}, \quad (1.1)$$

where  $H_{osc}$  denotes the Hubble parameter at the beginning of the oscillation. During the radiation-dominated epoch<sup>2</sup>, the ratio  $r$  grows and reaches

$$r \leq \frac{\sqrt{H_{osc} M_p}}{T_d} \frac{m_\sigma^2 \sigma_{osc}^2}{H_{osc}^2 M_p^2}, \quad (1.2)$$

provided the curvaton potential is dominated by a positive quadratic term. Here,  $T_d$  is the temperature for curvaton decay. The basic concept of the curvaton scenario is that the curvature perturbation

$$\zeta \simeq \frac{r}{3} \frac{\delta \rho_\sigma}{\rho_\sigma} \quad (1.3)$$

is generated by curvaton decay if the curvaton dominates the energy density of the Universe. The spectrum of the curvature perturbation related to the curvaton fluctuation is given by

$$\mathcal{P}_\zeta^{\frac{1}{2}} \simeq \frac{2r}{3} \frac{H_I}{2\pi\sigma_{osc}}, \quad (1.4)$$

where  $H_I$  denotes the Hubble parameter during inflation. Since the observational data requires  $\mathcal{P}_\zeta^{\frac{1}{2}} = 5 \times 10^{-5}$ , the curvaton scenario leads to the condition

$$\frac{2r}{3} \frac{H_I}{2\pi\sigma_{osc}} \simeq 5 \times 10^{-5}. \quad (1.5)$$

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<sup>2</sup>This radiation could be the radiation in a hidden sector.

Using (1.2) and (1.5), the condition for the cosmological parameters related to the curvaton scenario is given by

$$\frac{2}{3} \frac{H_I}{2\pi\sigma_{osc}} \frac{\sqrt{H_{osc}M_p}}{T_d} \frac{m_\sigma^2 \sigma_{osc}^2}{H_{osc}^2 M_p^2} \geq 5 \times 10^{-5}. \quad (1.6)$$

Here, the obvious bound from nucleosynthesis is  $T_d > 1$  MeV, which gives a bound

$$\frac{2}{3} \frac{H_I \sigma_{osc} m_\sigma^2}{2\pi H_{osc}^{\frac{3}{2}} M_p^{\frac{5}{2}}} \geq 5 \times 10^{-26}. \quad (1.7)$$

Alternatively, we may use a lower bound for the curvaton decay rate ( $\Gamma_\sigma \geq \frac{m_\sigma^3}{M_p^2}$ ) that leads to the condition  $T_d \simeq \sqrt{M_p \Gamma} \geq M_p (m_\sigma/M_p)^{\frac{3}{2}}$ . Thus, we obtain an alternative condition

$$\frac{2}{3} \frac{H_I \sigma_{osc} m_\sigma^{\frac{1}{2}}}{2\pi H_{osc}^{\frac{3}{2}} M_p^{\frac{5}{2}}} \geq 5 \times 10^{-5}. \quad (1.8)$$

The four parameters  $(H_I, \sigma_{osc}, H_{osc}, m_\sigma)$  that appear in the above calculations give a cosmological boundary condition that characterizes the model. Considering both (1.5) and  $r < 1$ , we find

$$\frac{2}{3} \frac{H_I}{2\pi\sigma_{osc}} > 5 \times 10^{-5}, \quad (1.9)$$

and then from Eq.(1.9) and (1.7) we find

$$H_I > 10^{-15} \times \frac{H_{osc}^{\frac{3}{4}} M_p^{\frac{5}{4}}}{m_\sigma}. \quad (1.10)$$

Alternatively, Eq.(1.9) and (1.8) give the condition

$$H_I > 10^{-4} \times \frac{H_{osc}^{\frac{3}{4}} M_p^{\frac{1}{2}}}{m_\sigma^{\frac{1}{4}}}. \quad (1.11)$$

In order to determine a clear bound for the inflation scale ( $H_I$ ), we need to fix the boundary condition. We assume that the curvaton mass ( $m_\sigma$ ) is constant during the period that we are interested in. Then, the oscillation of the curvaton field starts when the Hubble parameter falls below the curvaton mass (i.e., when the Hubble parameter reaches  $H = m_\sigma$ ), which leads to the boundary condition

$$H_{osc} \simeq m_\sigma. \quad (1.12)$$

Since  $H_I > H_{osc}$  always holds, we introduce a parameter  $p$  that measures the ratio between  $H_I$  and  $H_{osc}$ , defined as  $p^2 \equiv H_{osc}/H_I$ . Using this new parameter we can replace  $H_{osc}$  and

$m_\sigma$  in Eq.(1.10) with  $H_I$  and  $p$ , giving

$$H_I > M_p \times 10^{-12} \times p^{-\frac{2}{5}} \simeq 10^6 p^{-\frac{2}{5}} \text{GeV}. \quad (1.13)$$

From Eq.(1.11) we also find

$$H_I > 10^{-8} M_p p^2 \simeq 10^{10} p^2 \text{GeV}. \quad (1.14)$$

Note that the bound in Eq.(1.14) is effective only when  $p > 10^{-2}$ , which suggests that a tiny  $p \ll 1$  does not simply relax the bound. Therefore, the above conditions lead to a lower bound for the inflation scale;

$$H_I > 10^7 \text{GeV}. \quad (1.15)$$

This result is true for the traditional curvaton that satisfies the above boundary conditions.

To summarize, an alternative (maybe the curvaton) is needed for low-scale inflation to avoid problems in the generation of the curvature perturbation, but the bound in Eq.(1.15) suggests that the curvaton paradigm does not accommodate a low inflation scale. For the simplest curvaton, it is impossible to support low-scale inflation[3], even if careful fine-tuning is introduced to the theory. This is a critical problem for low-scale gravity signals that might be observed in the Large Hadron Collider(LHC).<sup>3</sup> The curvaton could be identified from the brane distance, bulk field or the flat direction of supersymmetric gauge theory, though we will not specify the origin of the curvaton in this paper. There are many model-dependent discussions for the curvaton hypothesis, but they are beyond the scope of this paper. Our goal in this paper is to relax the energy condition of Eq.(1.15) starting with the curvaton hypothesis so that the curvaton treatment can support low-scale inflation. There are many alternatives<sup>4</sup> that can be

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<sup>3</sup>If the fundamental scale is as low as the TeV scale, we must introduce additional symmetries or mechanisms to the phenomenological model so that unwanted interactions (such as baryon-number violating interactions) do not appear in the effective theory. Inflation with low-scale gravity is also a problem, which may or may not be solved by using the previous analyses[3]. However, the purpose of this paper is not to solve problems related to low-scale gravity, though low-scale gravity is a motivation for considering relaxation of the energy bound.

<sup>4</sup>The alternative formulations can be categorized as inhomogeneous reheating[4], inhomogeneous preheating[6],  $\delta N$  generation[5] or hybrids of these approaches[7]. The bound for the curvaton cannot be applied to these alternatives.

applied to this problem. We believe that it is worthwhile finding the simplest solution to this problem without introducing significant modifications to the traditional curvaton hypothesis. Arguments for the validity of the curvaton hypothesis are highly model-dependent.

## 1.1 Evolution on Concave potential

In this section we examine the bound given in Eq.(1.15) if  $\delta\sigma$  and  $\sigma$  evolve after inflation but before oscillation begins. Following the argument given in Ref.[8], the ratio  $\delta\sigma/\sigma$  is almost constant after inflation even if  $\delta\sigma$  and  $\sigma$  evolve after inflation. Therefore, for a concave potential  $V \simeq \frac{1}{2}m_\sigma^2\sigma^2$ , we introduce a parameter  $\gamma$  defined by

$$\begin{aligned}\delta\sigma(t_{osc}) &= \gamma\delta\sigma(t_{ini}) \\ \sigma(t_{osc}) &= \gamma\sigma(t_{ini}),\end{aligned}\tag{1.16}$$

where  $t_{osc}$  and  $t_{ini}$  denote the time when curvaton oscillation starts and the time when inflation ends, respectively.<sup>5</sup> We can use these relations to evaluate a new energy bound for  $\gamma \neq 1$ . Using  $\gamma$ , we find

$$H_I > 10^{-15} \times \frac{H_{osc}^{\frac{3}{4}} M_p^{\frac{5}{4}}}{m_\sigma \gamma}\tag{1.17}$$

rather than the expression given in (1.10). We also find

$$H_I > 10^{-4} \times \frac{H_{osc}^{\frac{3}{4}} M_p^{\frac{1}{2}}}{m_\sigma^{\frac{1}{4}} \gamma},\tag{1.18}$$

rather than (1.11). From these equations and the definition of  $p$ , we find

$$H_I > \gamma^{-1} 10^7 \text{GeV},\tag{1.19}$$

which is not relaxed by curvaton evolution after inflation if  $\gamma \ll 1$ . Therefore, for a concave potential, the bound of the energy scale is not relaxed by curvaton evolution. This result supports the condition  $H_I > 10^7 \text{ GeV}$  given in Ref.[2]. It was suggested in Ref.[2] that this condition can be relaxed by considering a “heavy curvaton” [2] that gives the non-standard boundary condition

$$m_\sigma(t_{osc}) \gg H(t_{osc}),\tag{1.20}$$

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<sup>5</sup>Note that the ratio  $\delta\sigma/\sigma$  is invariant during this evolution, and that the same result applies for the tachyonic evolution ( $\gamma > 1$ ) on a convex potential.

which is due to a phase transition at the end of inflation. This new boundary condition significantly alters the bound and gives a milder condition

$$H_I > 10^5 \text{GeV}. \quad (1.21)$$

However, this is not sufficient to support the inflation energy scale as low as  $H_I \sim O(1)\text{TeV}$ . As we have presented in Ref.[9], if the phase transition that gives a heavy curvaton occurs later than the primordial inflation, it may lead to another boundary condition that may remove the bound for  $H_I$ . Other solutions for this problem have been extensively studied[7] using other alternative formulations. In this paper we consider a generic case in which both the coefficient  $c(t)$  and the sign of the quadratic term depend on the specific conditions of the system.<sup>6</sup> We show that hilltop curvatons can significantly relax the condition for the inflation scale without introducing additional components or symmetries to the traditional curvaton hypothesis. Evolution of a curvaton on a convex potential that leads to tachyonic enhancement of the fluctuation has already been discussed for a specific model of Peccei-Quinn field curvatons[10]. In this model, amplification occurs when the curvaton is stabilized for a period near a maximum of the potential.<sup>7</sup> Although tachyonic amplification with static potential has already been discussed for some specific models, the generality of tachyonic enhancement is unclear for a non-static potential. We show that a hilltop curvaton, whose mass is not protected from modest  $O(H)$  corrections, can achieve significant enhancement of the curvaton fluctuation.<sup>8</sup>

## 1.2 Hilltop curvatons

We now discuss the enhancement of curvaton fluctuation with a convex potential without introducing additional fine-tuning to the traditional curvaton scenario. The evolution of the curvaton ( $\sigma$ ) and its fluctuation ( $\delta\sigma$ ) is given in terms of the effective potential

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<sup>6</sup>See appendix A for more details.

<sup>7</sup>See also the comment given in Ref.[2]. Another mechanism for this enhancement has been discussed in Ref.[11] for pseudo-Nambu-Goldstone curvatons, in which there is an additional direction of motion other than the curvaton.

<sup>8</sup>See Fig.(1) and (2) for the difference between tachyonic amplification on static and non-static potentials.

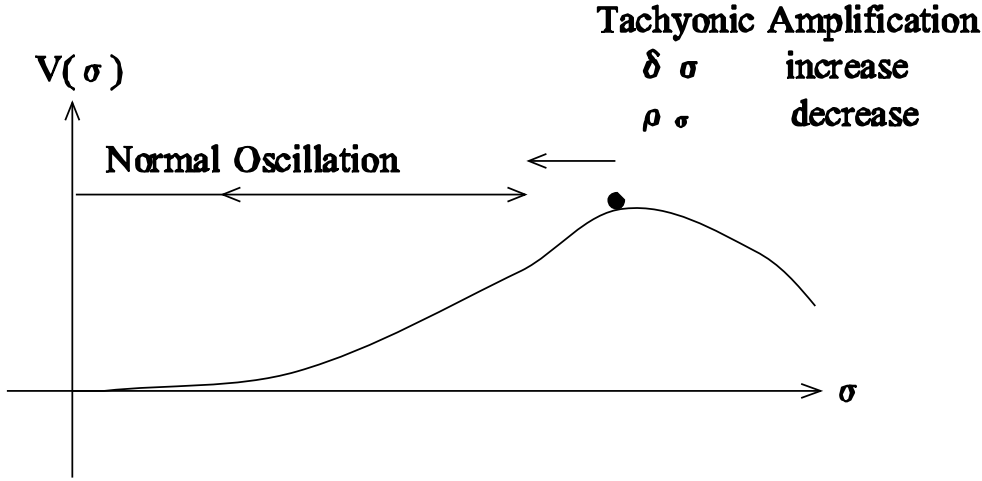


Figure 1: This figure shows tachyonic amplification of the curvaton fluctuation on a single-field static potential.  $\delta\sigma$  grows near the top of the potential, but the energy density of the curvaton  $\rho_\sigma$  decreases during this period. It is possible to enhance the ratio  $r$  if the radiation energy density diminishes significantly during this period. This is possible if there is “temporary rest”[10] of the curvaton near the maximum. Our mechanism is different from this amplification mechanism.

$(V(\sigma, H))$  by the equation

$$\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0, \quad (1.22)$$

where  $V_\sigma$  is the  $\sigma$ -derivative of the effective potential, and by the equation

$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + V_{\sigma\sigma}\delta\sigma = 0. \quad (1.23)$$

Here the dot denotes differentiation with respect to the cosmic time  $t$ . Following the original curvaton hypothesis, we consider an effective potential that represents the conventional curvaton potential except for the explicit  $O(H)$ -correction;

$$V = \frac{1}{2} [m_\sigma^2 - (cH)^2] \sigma^2 + \frac{g}{n!} \frac{\sigma^n}{M_*^{n-4}}, \quad (1.24)$$

where  $n$  is an integer ( $n > 4$ ), and the cut-off scale  $M_*$  is supposed to be  $M_* \simeq M_p$ .<sup>9</sup> Following the curvaton hypothesis, quartic term of the curvaton potential must be neg-

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<sup>9</sup>In some phenomenological models the cut-off scale  $M_*$  can be much smaller than the Planck scale. In that case symmetries or mechanisms are required to suppress the coefficients of higher dimensional interactions so that the curvaton potential remains flat. We follow the original curvaton hypothesis and do not discuss symmetries and mechanisms in this paper since they are highly model-dependent.

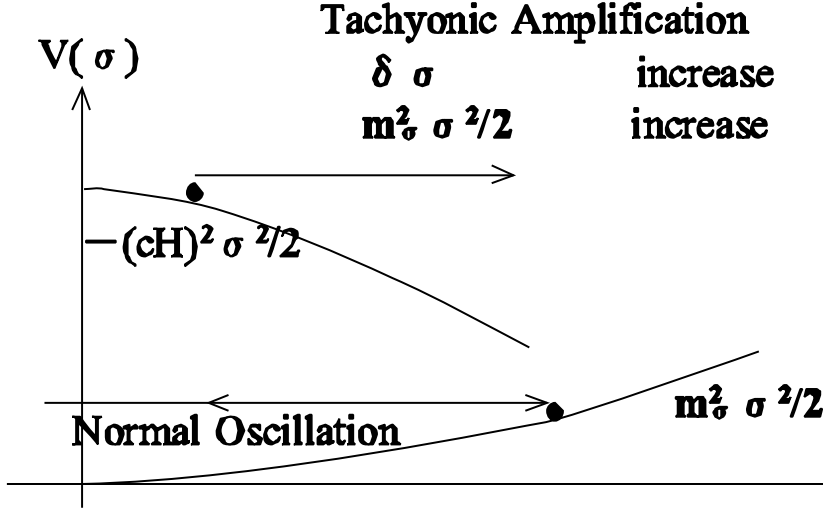


Figure 2: This figure shows tachyonic amplification of  $\delta\sigma$  on a single-field non-static potential. Note that both  $\delta\sigma$  and  $m_\sigma^2\sigma^2/2$  increases during this period.

ligible.<sup>10</sup> The evolution of the curvaton after  $cH < m_\sigma$  is precisely the same as the conventional scenario. Evolution before the curvaton oscillation is determined by the  $O(H)$ -correction term; in our model it leads to tachyonic enhancement. The sign of the Hubble-induced mass term depends on the conditions and must be negative during tachyonic enhancement. For simplicity, we fix the sign of the Hubble-induced mass term to be negative, as this assumption does not reduce the generality of the following argument. We assume that the expectation value of the curvaton  $\sigma_0 \simeq 10^5 \times \delta\sigma_0$  is always smaller than the effective minimum until  $m_\sigma$  dominates the curvaton mass. Note that if oscillation occurs about the effective minimum, the curvaton fluctuation is reduced. Then, the evolution of the curvaton during inflation is given by

$$\sigma \simeq \sigma_0 e^{F\Delta N}, \quad (1.25)$$

where

$$F \equiv \sqrt{\frac{9}{4} + c^2} - \frac{3}{2}. \quad (1.26)$$

This is the so-called fast-roll evolution[12]. Comparing (1.22) and (1.23), it is straightforward to derive the equation for  $\delta\sigma$  as

$$\delta\sigma \simeq \delta\sigma_0 e^{F\Delta N}. \quad (1.27)$$

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<sup>10</sup>There can be many terms that are neglected in the above expression for the curvaton potential. Higher dimensional terms may have either negative or positive sign.



Since the spectrum index of the curvaton perturbation is given by  $n - 1 \simeq 2\eta_\sigma$  for models with negligible slow-roll parameter  $\epsilon_H$ , we take the condition-dependent coefficient to be  $c \simeq 0.2$  during inflation.<sup>11</sup> Here the definition of the slow-roll parameter  $\eta_\sigma$  is

$$\eta_\sigma \equiv \frac{M_p^2 V_{\sigma\sigma}}{V_I}. \quad (1.28)$$

Therefore, the exponential factors in the above equations are at most  $e^{F\Delta N} \simeq e^{0.017 \times 60} < e$ . Thus, we conclude that the evolution of the curvaton and its fluctuation during inflation is not significant.

The curvaton dynamics after inflation are different from those of the inflationary epoch. We introduce the barotropic parameter ( $w$ ) of the dominant fluid, which does not relate to the curvaton, but instead relates to the background radiation ( $w = 1/3$  for radiation) or the inflaton oscillation (for inflaton oscillation  $w$  depends on the order of the inflaton potential[13]). As we are considering the evolution of the curvaton before the beginning of curvaton oscillation at  $m_\sigma \sim H_{osc}$ , the dominant part of the effective potential is

$$V \simeq -\frac{1}{2}(cH)^2\sigma^2. \quad (1.29)$$

Here, both  $H$  and  $\sigma$  depend on the cosmic time  $t$ , while  $c$  is a constant determined by the dominant component of the Universe. The value of  $c$  during inflaton oscillation is believed to be the same as that of the inflationary epoch, while  $c$  may be different during the radiation-dominated epoch.<sup>12</sup> Substituting the  $t$ -derivative by  $dt \equiv (1 + w)d\tau$ , we find[8]

$$\sigma'' + (1 - w)\sigma' + \frac{4c}{9}\sigma = 0, \quad (1.30)$$

where the prime denotes differentiation with respect to  $\tau$ . Solving the above equation for  $\tau$  and then substituting  $t$ , we find[8]

$$\sigma(t_{out}) \simeq \sigma(t_{in}) \left( \frac{t_{out}}{t_{in}} \right)^{-\frac{1}{2} \frac{K}{1+w}}, \quad (1.31)$$

where  $K$  is defined as

$$K \equiv (1 - w) - \sqrt{(1 - w)^2 + \frac{16}{9}c^2}. \quad (1.32)$$

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<sup>11</sup>On the other hand, if the slow-roll parameter  $\epsilon_H$  is responsible for the current observational value of the spectral index  $n$ , the value of  $c$  during inflation is not determined by  $n$ . In this case, there is an upper bound  $c < 0.2$  if there is no fine-tuning between  $\epsilon_H$  and  $\eta_\sigma$ .

<sup>12</sup>See appendix for more details.

Here,  $t_{in}$  and  $t_{out}$  are the time at the beginning and end of the evolution, respectively. The evolution of the curvaton and its fluctuation is negligible for  $c \simeq 0.2$ , while it is significant for  $c > 1$ . Since the potential becomes concave near the minimum of the effective potential, we consider the evolution within  $\sigma(t_{out}) < \sigma_{min}$ , where  $\sigma_{min}$  denotes the effective minimum for  $m_\sigma < cH$ . As the time passes, the bare mass ( $m_\sigma$ ) starts to dominate the potential and tachyonic amplification stops at  $t = t_{out}$ , ( $H(t_{out}) \simeq m_\sigma$ ). Then, the curvaton oscillation starts at  $t = t_{out}$ , as for a traditional curvaton.

We now consider a concrete example. We consider a case in which the coefficient of the induced mass is as large as  $c = 5$  after reheating. Reheating occurs at  $t = t_{in}$ , then the curvaton oscillation starts at  $t = t_{out} = t_{osc}$ . With this simple and generic situation we examine the energy condition for the hilltop curvaton. We introduce a new parameter  $q$ , which gives the reheating temperature

$$T_R \simeq 10^{q/2} m_\sigma. \quad (1.33)$$

Using this parameter, the ratio  $t_{out}/t_{in}$  is given by  $t_{out}/t_{in} = 10^q$ . Considering Eq.(1.31) and (1.32), we find the enhancement factor

$$\gamma \simeq 10^{\frac{1}{2}qc}. \quad (1.34)$$

For  $q = 2$ , the enhancement factor  $\gamma$  is given by  $\gamma \simeq 10^5$ . Note that the condition  $q < -\log(H_I/H_{osc}) \equiv -\log(p^2)$  applies because of the relation  $t_I < t_{in} < t_{out} < t_{osc}$ , where  $t_I$  denotes the time when inflation ends. Since (1.19) is true for  $p = 10^{-2}$ , we find

$$H_I > 10^2 \text{GeV} \quad (1.35)$$

for  $c = 5$ ,  $q = 2$  and  $p = 10^{-2}$ . In this case, we find  $\sigma(t_{in}) > 10^7 \text{GeV}$  and  $\sigma(t_{out}) > 10^{12} \text{GeV}$ . The required condition for the effective potential is that  $\sigma(t_{out})$  is smaller than  $\sigma_{min}$  during the tachyonic amplification.

This significant reduction of the inflation energy is a generic consequence of the enhancement of the curvaton fluctuation induced by the  $O(H)$  mass. The required condition for this amplification are (1) if the coefficient of the Hubble-induced mass ( $c$ ) is somewhat large during the radiation-dominated epoch and (2) the sign of the mass term is negative, then the curvaton perturbation grows significantly on a convex potential during the

radiation-dominated epoch. This significant enhancement gives a significant relaxation of the energy condition for the primordial inflation.

## 2 Conclusions and Discussions

We have studied a scenario for hilltop curvatons in which a curvaton evolves on a convex potential. We assumed that the effective mass of a curvaton is expressed as  $m_\sigma = cH$ , where the coefficient varies within  $0.2 \leq c \leq 5$  depending on the dominant component of the Universe. A negative mass term leads to enhancement of the curvaton fluctuation. In our model, both  $\delta\sigma$  and the ratio  $r$  at the beginning of the oscillation are enhanced during this epoch, which has a significant impact on the energy bound for low-scale inflation. Following the curvaton hypothesis, we have demonstrated the generality of this enhancement. The Hubble-induced mass is necessary for the significant enhancement of the curvaton perturbation in our model. If there is no  $O(H)$  correction to the curvaton potential, our scenario requires two different kinds of phase transitions that change the effective mass of the curvaton at least twice. These phase transitions must occur (1) between inflation and tachyonic enhancement and (2) between tachyonic enhancement and curvaton oscillation. Moreover, tachyonic enhancement with a constant mass is very dangerous for our scenario because a curvaton will soon start to oscillate about the effective minimum. In order to avoid the reduction of  $\delta\sigma$  during this oscillation, the second phase transition must take place soon after the first. The important idea of our scenario is that for a  $c \sim O(1)$  potential, the tachyonic amplification lasts long enough that the conventional curvaton oscillation may start before the beginning of the oscillation about the effective minimum.

In addition to the condition required for the tachyonic enhancement, we must consider the origin of the spectral index  $n \neq 1$ . For the curvaton, this index is given by

$$n - 1 \simeq 2\eta_\sigma - 2\epsilon_H, \quad (2.1)$$

where  $\epsilon_H \equiv -\dot{H}/H^2$  is the slow-roll parameter. In many models of inflation in which  $\epsilon_H$  is very small,  $\eta_\sigma$  must be responsible for  $n \neq 1$ . However, if there is no mechanism that forces  $|\eta_\sigma| \simeq 0.02$  (i.e.,  $m_\sigma \sim O(H)$ ) during inflation, it is very difficult to attain the

present observational value of the spectral index without introducing fine-tuning. This may reduce the generality of the curvaton hypothesis, narrowing the range of possible inflation models in which the curvaton can explain the curvature perturbation.<sup>13</sup> In this sense, the  $O(H)$  correction is important for the generality of the curvatons, even when tachyonic enhancement does not play an important role. It should be noted again that the required condition for the curvature perturbation is very severe for low-scale inflation[3] if it is generated by the inflaton fluctuation.

### 3 Acknowledgment

We wish to thank K.Shima for encouragement, and our colleagues at Tokyo University for their kind hospitality.

### A $O(H)$ -corrections

The purpose of this appendix is to show how  $O(H)$  corrections to light scalar fields appear in the radiation-dominated epoch. We will mainly follow the original argument given by Dine et.al, in Ref.[15]. The supergravity scalar potential is given by

$$V = e^{\frac{K}{M_p^2}} \left[ (D_i W) K^{i\bar{j}} (D_{\bar{j}} W)^* - \frac{3}{M_p^2} |W|^2 \right] + V_D, \quad (\text{A.1})$$

where  $W$  and  $K$  are the superpotential and Kähler potential, and the derivatives are defined as  $D_i W = W_i + K_i W/M_p^2$  and  $W_i = \partial W/\partial \phi_i$ . Looking at this scalar potential, there is a  $O(H)$ -correction appears for the soft mass of a light field  $\sigma$  when the energy density is dominated by  $F$ -terms. This is the most discussed origin of the  $O(H)$ -corrections. However in the original argument the origin of such terms is not confined to the scalar potential. Following Ref.[15], we consider  $O(H)$  corrections that may appear from nonrenormalizable interactions such as

$$\delta K \propto \Sigma^\dagger \Sigma \Phi^\dagger \Phi / M_p^2, \quad (\text{A.2})$$

where the scalar component of the superfield  $\Sigma$  is the light field  $\sigma$ , and  $\Phi$  is a field that dominates the energy density of the Universe. Then, the energy density of the Universe

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<sup>13</sup>See also Ref.[14]

is given by  $\rho \simeq \int d^4\theta \Phi^\dagger \Phi$ . Note that in the thermal phase, the expectation value arises from kinetic terms, while the usual  $\delta m_\sigma^2 \simeq |F_\Phi|^2/M_p^2$  correction is significant in a  $F_\Phi$ -dominated Universe. Following this argument, a  $O(H)$  correction may result even if D-term dominates the potential.

From the above argument it is clear that there is no reason to assume  $c$  in  $\delta m_\sigma = cH$  takes exactly the same value in both the inflation stage and the thermal phase. This is the main reason why we considered  $O(1)$  variation to  $c$ .

## References

- [1] A. D. Linde and V. F. Mukhanov, *Nongaussian isocurvature perturbations from inflation*, *Phys.Rev.D*56(197)535 [astro-ph/9610219]; D. H. Lyth, D. Wands, *Generating the curvature perturbation without an inflaton*, *Phys.Lett.B*524(2002)5 [hep-ph/0110002]; T. Moroi, T. Takahashi, *Effects of Cosmological Moduli Fields on Cosmic Microwave Background*, *Phys.Lett.B*522(2001)215 [hep-ph/0110096].
- [2] D. H. Lyth, *Can the curvaton paradigm accommodate a low inflation scale?*, *Phys.Lett.B*579(2004)239-244,2004[hep-th/0308110].
- [3] D. H. Lyth, *Constraints on TeV scale hybrid inflation and comments on nonhybrid alternatives*, *Phys.Lett.B*466(1999)85 [hep-ph/9908219]; A. M. Green and A. Mazumdar, *Dynamics of a large extra dimension inspired hybrid inflation model*, *Phys.Rev.D*65(2002)105022 [hep-ph/0201209]; R. N. Mohapatra, A. Perez-Lorenzana, and C. A. de S. Pires, *Inflation in models with large extra dimensions driven by a bulk scalar field*, *Phys.Rev.D*62(2000)105030 [hep-ph/0003089]; T. Matsuda, *Kaluza-Klein modes in hybrid inflation*, *Phys.Rev.D*66(2002)107301 [hep-ph/0209214]; T. Matsuda, *Nontachyonic brane inflation*, *Phys.Rev.D*67(2003)083519 [hep-ph/0302035]; A. Mazumdar, *New developments on embedding inflation in gauge theory and particle physics* [arXiv:0707.3350].
- [4] G. Dvali, A. Gruzinov and M. Zaldarriaga, *A new mechanism for generating density perturbations from inflation*, *Phys.Rev.D*69(2004)023505 [astro-ph/0303591];

- F. Vernizzi, *Cosmological perturbations from varying masses and couplings*, *Phys.Rev.D*69(2004)083526 [astro-ph/0311167].
- [5] D. H. Lyth, *Generating the curvature perturbation at the end of inflation*, *JCAP* 0511:006,2005 [astro-ph/0510443]; F. Bernardeau, L. Kofman and J.P. Uzan, *Modulated fluctuations from hybrid inflation*, *Phys.Rev.D*70(2004)083004 [astro-ph/0403315]; T. Matsuda, *Elliptic Inflation: Generating the curvature perturbation without slow-roll*, *JCAP* 0609:003,2006 [hep-ph/0606137]; T. Matsuda, *Brane inflation without slow-roll*, *JHEP* 0703:096,2007 [astro-ph/0610402].
- [6] E. W. Kolb, A. Riotto and A. Vallinotto, *Curvature Perturbations from Broken Symmetries*, *Phys.Rev.D*71(2005)043513[astro-ph/0410546]; T. Matsuda, *Generating the curvature perturbation with instant preheating*, *JCAP* 0703:003,2007 [hep-th/0610232]; T. Matsuda, *Generating curvature perturbations with MSSM flat directions*, *JCAP* 0706:029,2007[hep-ph/0701024].
- [7] T. Matsuda, *Topological curvatons*, *Phys.Rev.D*72(2005)123508 [hep-ph/0509063]. T. Matsuda, *NO Curvatons or Hybrid Quintessential Inflation*, *JCAP* 0708:003,2007 [arXiv:0707.1948]; T. Matsuda, *Cosmological perturbations from inhomogeneous preheating and multi-field trapping*, *JHEP* 0707:035,2007 [arXiv:0707.0543]; T. Matsuda, *Hybrid curvatons from broken symmetry*, *JHEP* 0709:027,2007 [arXiv:0708.4098].
- [8] K. Dimopoulos, G. Lazarides, D. Lyth and R. R. de Austri, *Curvaton dynamics*, *Phys.Rev.D*68(2003)123515 [hep-ph/0308015].
- [9] T. Matsuda, *Curvaton paradigm can accommodate multiple low inflation scales*, *Class.Quant.Grav.*21(2004)L11 [hep-ph/0312058]; K. Dimopoulos, D. H. Lyth and Y. Rodriguez, *Low scale inflation and the curvaton mechanism*, *JHEP* 0502:055,2005 [hep-ph/0411119]; Y. Rodriguez, *Low scale inflation and the immediate heavy curvaton decay*, *Mod.Phys.Lett.A*20:2057,2005 [hep-ph/0411120].
- [10] K. Dimopoulos, G. Lazarides, D. Lyth and R. R. de Austri, *The Peccei-Quinn field as curvaton*, *JHEP* 0305:057,2003 [hep-ph/0303154].

- [11] K. Dimopoulos, D. H. Lyth, A. Notari and A. Riotto, *The Curvaton as a pseudoNambu-Goldstone boson*, *JHEP* 0307:053,2003 [hep-ph/0304050].
- [12] A. Linde, *Fast roll inflation*, *JHEP* 0111:052,2001 [hep-th/0110195].
- [13] M. S. Turner, *Coherent Scalar Field Oscillations in an Expanding Universe*, *Phys.Rev.D*28(1983)1243.
- [14] L. Boubekur and D. H. Lyth, *Hilltop inflation*, *JCAP* 0507:010,2005[hep-ph/0502047].
- [15] M. Dine, L. Randall, S. D. Thomas, *Supersymmetry breaking in the early universe*, *Phys.Rev.Lett.*75(1995)398 [hep-ph/9503303].